4. Drawing on Physical Analogies

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Graph layout methods described in previous chapters were based on structural characteristics of the graph, or a preprocessed version of the graph. Often, such knowledge is not provided. In this chapter, we take a look at a class of methods applicable to general graphs, without prior knowledge of any structural properties. Their common denominator is that they liken the graph to a system of interacting physical objects, the underlying assumption being that relaxed (energy-minimal) states of suitably defined systems correspond to readable and informative layouts.

Methods based on physical analogies are quite popular, mainly for three reasons. First of all, they are very intuitive, because layout is related to the everyday experience of the surrounding physical world. Secondly, their basic instances are comparatively easy to understand and to program. The threshold to get started is thus very low. And finally, they often yield fairly satisfactory results on medium-sized graphs up to around 50 vertices. In general, these methods consist of two components,

- a *model* consisting of physical objects (representing the elements of the graph) and interactions between these objects, and
- an *algorithm* that (approximately) computes an equilibrium configuration of the system.

The specifications of a model fully represent the intuition behind what is considered a good layout, ideally depending on the specific context of the graph. Its associated algorithm merely serves as an optimization routine for the objective function explicated in the model.

First we introduce the fundamental concept behind physical modeling, that is the basis for all methods presented in this chapter. Then we describe a number of models and associated algorithms in Sections 4.2 and 4.3. An asset of physical modeling that is often overlooked is its inherent flexibility. For this reason, we conclude this chapter by listing examples of model specifications tailored to specific layout objectives.

4.1 The Springs

Given a connected undirected graph with no particular background information, the following two criteria of readable layout seem to be generally agreed upon for the conventional two-dimensional straight-line representation.

1. Vertices should spread well on the page.
2. Adjacent vertices should be close.

Only intuitive explanations can be offered. While uniform vertex distribution reduces clutter, the implied uniform edge lengths leave an undistorted impression of the graph. Since "clutter" and "distortion" already have physical connotations, it seems fairly natural to start thinking of a more specific physical analogy.

We are used to observing even spacing between repelling objects. This makes it natural to imagine vertices behaving like charged balls to satisfy the first criterion. A physical analogy for the second criterion is also easy to find, since it states that we should not allow adjacent balls to drift too far apart. Springs replacing edges will do the job. Springs are better suited than, for example, sticks or ropes, because they can be both extended and compressed to allow moderate distortion, but exert increasingly strong forces when deviating further from their natural length. Moderate distortion may be inevitable, since it is impossible to represent every graph with straight edges of equal length. Note that it is $\mathcal{NP}$-hard to decide whether an arbitrary graph has a straight-line embedding with equal edge lengths in any number of dimensions (Johnson, 1982) or just any planar straight-line embedding (Eades and Wormald, 1990).

Figure 4.1 illustrates the imaginary substitution of vertices and edges with charged balls and connecting springs, respectively. If the system is let go, it attains an equilibrium state in which all forces cancel each other, and the substitution can be reversed to obtain a straight-line drawing that satifies the criteria from above at least approximately.

Formally, such a model can be expressed either in terms of forces acting on the physical objects, or in terms of a potential energy reflecting the internal stress of the system and thus describing how well a configuration matches the design goals modeled in the system. Algorithms to simulate a system's relaxation typically try to move the objects iteratively into stable states in which all forces cancel each other, or to minimize the energy directly. Prominent examples from each kind of formalization are described in the following two sections. In Section 4.4, we give examples for creative use of these analogies in more advanced layout models that include various criteria for good layout.

4.2 Force-Directed Placement

The seminal paper for physical modeling in graph drawing is a short text of Eades (1984), though closely related methods had already been described in the context of VLSI design (Fisk et al., 1967; Quinn and Breuer, 1979). Given a connected undirected graph $G = (V, E)$, let $p = (p_v)_{v \in V}$ be a vector of vertex positions $p_v = (x_v, y_v)$ in the plane. We denote by $\|p_v - p_u\|$ the length of the difference vector $p_v - p_u$, which is the Euclidean distance between positions p_u and p_v. Furthermore, we denote by $\overrightarrow{p_u p_v}$ the unit length vector $\frac{p_v - p_u}{\|p_v - p_u\|}$ pointing from p_u to p_v. The model of Eades (1984), now known as the

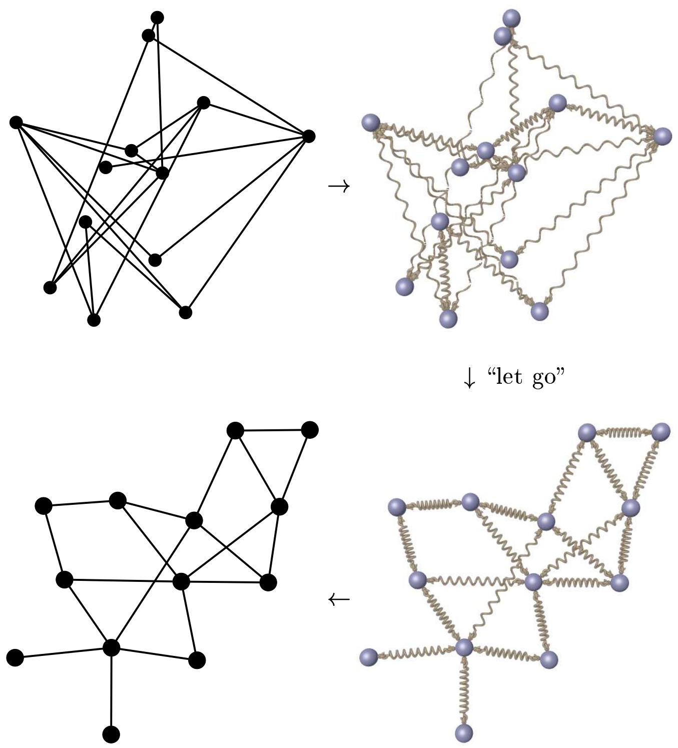


Fig. 4.1. The spring analogy.

spring embedder, implements the analogy described in the previous section. It is defined using repelling forces

$$f_{\mathsf{rep}}(p_u, p_v) = \frac{c_\varrho}{\|p_v - p_u\|^2} \cdot \overrightarrow{p_u p_v}$$

between every pair of non-adjacent vertices $u, v \in V$, where c_ϱ is a repulsion constant. Complementary spring forces between adjacent vertices $u, v \in V$ shall keep these sufficiently apart, yet close to each other. However, instead of more realistic forces according to Hooke's law, (imaginary) logarithmic springs which exert weaker forces on far apart vertices are employed. They yield forces

$$f_{\mathsf{spring}}(p_u, p_v) = c_\sigma \cdot \log \frac{\|p_u - p_v\|}{l} \cdot \overrightarrow{p_v p_u},$$

so that the direction depends on whether the actual distance is less or greater than a natural length l of the spring. Constant c_σ is a parameter controlling the strength of the spring. Figure 4.2 gives a qualitative impression of the forces a vertex u exerts on vertex v, depending on the distance between the two. The solid line shows the force in case u is adjacent to v (f_{spring}), while the dotted line indicates the force in case u is not adjacent to v (f_{rep}). Positive values signify a force dragging v towards u, whereas negative values signify a force repelling v from u.

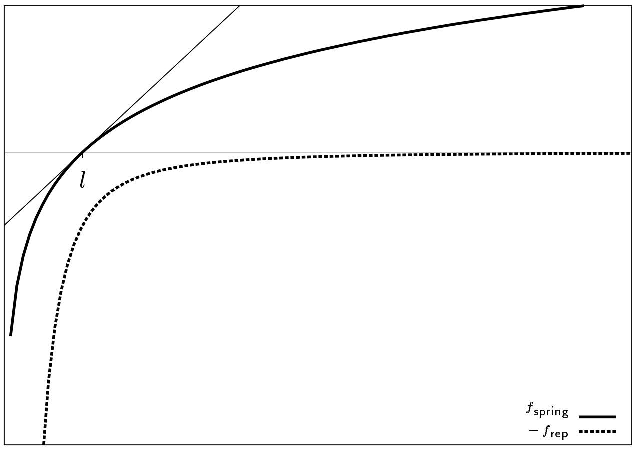


Fig. 4.2. Magnitude of spring embedder forces.

Next we address the question of how to obtain an equilibrium configuration. Vertex positions not corresponding to a system at equilibrium imply positive internal stress. To relax a stressed system, vertices are iteratively moved, at time t, according to a *net force vector* $F_v(t)$, which is the sum of all repulsion and spring forces acting on v. After computing $F_v(t)$ for all $v \in V$, each vertex is moved a constant δ times this vector. This constant is used to prevent excessive movement due to synchronous update. By iteratively computing the forces on all vertices and updating positions accordingly, the system approaches a stable state, in which no local improvement is possible. See Algorithm 6 for a concise description.

Algorithm 6: Spring embedder

Input: connected undirected graph $G = (V, E)$
initial placement $p = (p_v)_{v \in V}$

Output: placement p with low internal stress

for $t \leftarrow 1$ ***to*** ITERATIONS **do**
  for $v \in V$ **do**
    $F_v(t) \leftarrow \sum_{u : \{u,v\} \notin E} f_{\mathsf{rep}}(p_u, p_v) + \sum_{u : \{u,v\} \in E} f_{\mathsf{spring}}(p_u, p_v)$
  for $v \in V$ **do** $p_v \leftarrow p_v + \delta \cdot F_v(t)$

Despite its simplicity, the spring embedder produces satisfactory output in many cases. To even out some shortcomings of the method, several refinements have been developed. These refinements mainly aim at faster computation, but sometimes also at improved quality of the layout.

A number of heuristics is used by Fruchterman and Reingold (1991) to speed up many aspects of layout computation. Firstly, the forces are modified to allow faster evaluation. Repelling forces

$$f_{\mathsf{rep}}(p_u, p_v) = \frac{l^2}{\|p_u - p_v\|} \cdot \overrightarrow{p_u p_v}$$

are used between every pair of vertices, and additional attracting forces

$$f_{\mathsf{attr}}(p_u, p_v) = \frac{\|p_u - p_v\|^2}{l} \cdot \overrightarrow{p_v p_u}$$

are used between adjacent vertices. The combination of attraction and repulsion between adjacent vertices yields a spring-like force $f_{\mathsf{spring}}(p_u, p_v) = f_{\mathsf{attr}}(p_u, p_v) + f_{\mathsf{rep}}(p_u, p_v)$, similar in effect to the force used by Eades (1984). Since its magnitude increases more than proportionally with the distance (see Figure 4.3 for a comparison), one may also hope for faster convergence.

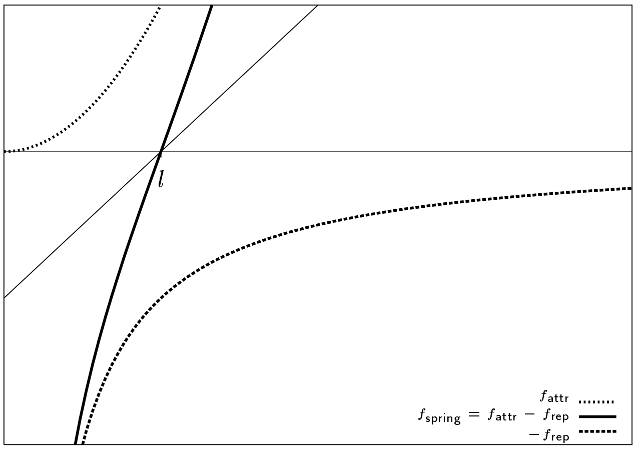
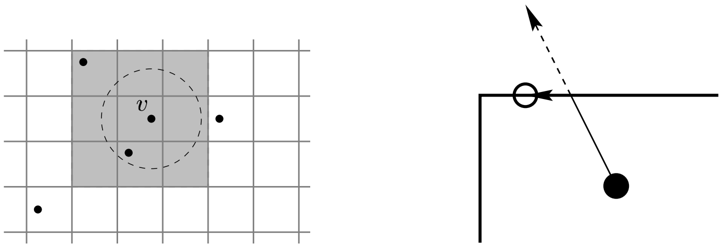


Fig. 4.3. Modified forces by Fruchterman and Reingold (1991).

A second heuristic to speed up computation does not change the objective function, but the precision of evaluation. Since repulsion from far away vertices does not contribute much to the displacement vector, such irrelevant vertices are omitted in the sum of repulsive forces using a grid technique. Only vertices lying in grid cells close to the cell of v are considered, and only if their distance is below a fixed threshold, a repulsive force is calculated and included in the sum of forces. See Figure 4.4(a).

Two other modifications with respect to Algorithm 6 are concerned with the displacement vector. Instead of applying a constant damping factor δ to the net force vector, the net force vector is clipped at a time-dependent maximum displacement $\delta(t)$ to prevent excessive changes, especially in later stages of the iteration when the placement is close to a stable state. The second modification to the displacement ensures that the graph is laid out inside of a given rectangular area, like a screen or a sheet of paper. If the displacement would position a vertex beyond a fixed boundary, the corresponding coordinate of the displacement vector is clipped.



(a) neglecting weak repulsive forces (b) coordinate clipping

Fig. 4.4. Spring embedder modifications of Fruchterman and Reingold (1991).

Another notable refinement of the basic spring embedder is described in Frick et al. (1995). Again, both forces and iteration scheme are modified to speed up the algorithm and to improve layout quality (under the same criteria). Repulsive and attractive forces are defined so that no square root has to be taken,

$$f_{\mathsf{rep}}(p_u, p_v) = \frac{l^2}{||p_u - p_v||^2} \cdot (p_u - p_v),$$
$$f_{\mathsf{attr}}(p_u, p_v) = \frac{||p_u - p_v||^2}{l^2 \cdot \Phi(v)} \cdot (p_v - p_u),$$

and all computations are performed using integer arithmetic. The denominator in the attractive force is defined as $\Phi(v) = 1 + \frac{d_G(v)}{2}$ and effectively slows down high-degree vertices. However, a new gravitational force is introduced, dragging each vertex towards the barycenter $\zeta = \sum_{w \in V} p_w$ of all vertices by

$$f_{\mathsf{grav}}(p_u, p_v) = \Phi(v) \cdot \gamma \cdot \left(\frac{\zeta}{|V|} - p_v \right),$$

which is stronger for high degree vertices. The contribution of this force is controlled via a gravitational constant γ. Similar to layout area restriction, gravitational forces prevent components of a disconnected graph from drifting arbitrarily far apart. In addition to the above forces, a small random force is added to the net force to make the algorithm more robust against poor equilibrium states.

To reduce the number of iterations, the net force vector of each vertex during the previous iteration, $F_v(t-1)$, is stored and compared to the current one. Each vertex has its own adaptive absolute displacement distance $\delta_v(t)$ that is modified according to the angular difference $\alpha = \angle(F_v(t-1), F_v(t))$ of the current and the previous net force vector. If a vertex is to be moved into roughly the same direction as before ($\sin\alpha \approx 1$), $\delta_v(t)$ is chosen larger, if it is to be moved in the opposite direction (*oscillation*: $\sin\alpha \approx -1$), $\delta_v(t)$ is chosen smaller. Like oscillation, *rotation* is an indicator of ineffective movement of a vertex. A *skew gauge* is updated, if the current and the previous displacement vector are almost perpendicular ($\cos\alpha \approx 1$), and $\delta_v(t)$ is lowered, if a large skew suggests that a vertex rotates around some position. During each iteration, vertices are visited in random order, and each position is updated immediately by $p_v \leftarrow p_v + \max\{\delta_{\mathsf{max}}, \delta_v(t)\} \cdot \frac{F_v(t)}{\|F_v(t)\|}$, where δ_{max} is a fixed maximum displacement. Our own experience confirms that these heuristics substantially reduce the number of iterations needed to reach a stable state.

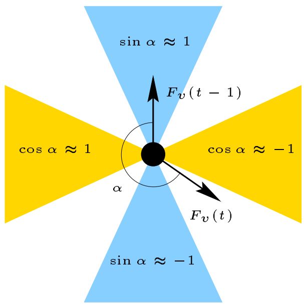


Fig. 4.5. Detection of oscillation and rotation (Frick et al., 1995).

The modifications of Fruchterman and Reingold (1991) and Frick et al. (1995) demonstrate that the spring embedder can be varied in many ways without changing its principle behavior. Clearly, many other heuristics are conceivable, but the two examples presented seem to cover sufficiently many aspects relevant to faster computation, faster convergence, and robust results.

In the next section, we turn to methods that try to satisfy the same criteria, but take a formally different approach by defining an explicit objective function.

4.3 Energy-Based Placement

Forces defined in the spring embedder variants described above indicate in which direction a vertex can be moved to reduce the forces acting on it, and thus an implicit internal energy of the physical system. Instead of displacing vertices according to these forces, one might as well attempt to minimize this energy directly. A spring of natural length l and of strength c_σ with actual length d (assumed to be within reasonable limits) has a *potential energy* of

$$U_{\mathsf{spring}}(d) = c_\sigma \cdot (d - l)^2,$$

Kamada and Kawai (1989) avoid a second potential for repulsion by using springs of different length and strength between every pair of vertices. Their specific choice of springs is governed by the assumption that the ideal distance between two vertices is the length of a shortest path between them, multiplied by the ideal length of a single edge, i.e. every path in the graph is best represented by a straight line. The natural length of the spring connecting vertices $u, v \in V$ is therefore chosen proportional to $d_G(u, v)$, which denotes the length of a shortest path between them. Clearly, perfect relaxation of all springs is impossible for most graphs, so local distances are rendered more important by using springs of strength inverse to their length. The resulting objective function is the sum over the potential energies of all $n \cdot (n-1)/2$ springs,

$$U_{\mathsf{KK}}(p) = \sum_{u,v \in V} \frac{c}{d_G(p_u, p_v)^2} \cdot \left(\|p_u - p_v\| - l \cdot d_G(u,v)\right)^2 ,$$

where c is a scaling constant, and l is the ideal length of a single edge.

To obtain a local minimum of this objective function, a modified Newton-Raphson method is applied. In a local minimum, all partial derivatives of U_{KK} are zero. This condition can be expressed in a system of dependent non-linear equations. Similar to Quinn and Breuer (1979), the Newton-Raphson method is modified in that the coordinates of a single vertex are updated while all others are fixed. In each iteration, the vertex with the longest gradient is picked and moved several times until its gradient falls below a given threshold.

It is interesting to note that the physically inspired objective function U_{KK} is closely related to the objective function

$$U_{\mathsf{MDS}}(p) = \frac{1}{\sum\limits_{u,v \in V} d_G(u,v)^2} \cdot \sum_{u,v \in V} \left(\|p_u - p_v\| - l \cdot d_G(u,v)\right)^2$$

of multidimensional scaling defined in (Kruskal and Wish, 1978). The family

$$S_k(p) = \frac{1}{\sum\limits_{u,v \in V} l_{u,v}^{2-k}} \cdot \sum_{u,v \in V} \frac{1}{l_{u,v}^k} \cdot \left(\|p_u - p_v\| - l_{u,v}\right)^2$$

of objective functions, where $l_{u,v}$ is the desired distance between vertices u and v, and $k \in \{0, 1, 2\}$, is discussed by Cohen (1997). While S_0 corresponds to multidimensional scaling, S_2 corresponds to the above layout objective function. As early as in the 1960s, multidimensional scaling was the first technique for automatic layout of social networks,[1] and it is still in use (Krackhardt et al., 1994). It seems, though, that the bias of U_{KK} towards exact representation of short distances, due to stronger short springs, results in layouts that display less clutter and have fewer small angles.

While the optimization method used by Kamada and Kawai (1989) does not differ notably from the ones in the previous section, the following simplification of the objective function admits exact optimization in time that is polynomial in the number of vertices. Instead of springs of some varying length, Tutte (1963) uses springs of ideal length zero. Setting the partial derivatives of the resulting objective function

$$U_{\mathsf{center}}(p) = \sum_{\{u,v\} \in E} \|p_u - p_v\|^2$$

equal to zero yields two independent systems of linear equations (one for each coordinate), as opposed to the one non-linear system obtained from U_{KK}. These linear systems of equations can conveniently be written in the form

$$(D - A) \cdot x = \mathbf{0},$$
$$(D - A) \cdot y = \mathbf{0},$$

where A is the adjacency matrix of G, D is the diagonal matrix of vertex degrees, and x and y are the vectors of x- and y-coordinates. The famous Matrix Tree Theorem (Kirchhoff, 1847) then states that the determinant of any submatrix obtained from $D - A$ by deleting a positive number of rows and their corresponding columns equals the number of spanning trees of the graph obtained from G by contracting the vertices corresponding to these rows into a new one. See Chaiken and Kleitman (1978) for several variants of this theorem. Since this number clearly is positive for connected graphs, so is the determinant, which implies that there is a unique layout minimizing U_{center}, if only the position of at least one vertex in each connected component of the input graph is fixed. This optimal layout can be computed by solving the smaller system of linear equations obtained by adjusting the right hand sides using the coordinates of fixed vertices.

Optimal layouts with respect to U_{center} are called *barycentric*, because the optimality conditions imply that every vertex not fixed in advance is placed in the barycenter of its neighbors. The main theorem of Tutte (1963) assures even more: barycentric layouts of 3-connected planar graphs are planar with

[1] Charles Kadushin, personal communication (1999).

all internal faces convex, if the vertices of a single face in the unique planar embedding are fixed to lie on a convex polygon (in appropriate order). Since the adjacency matrices of planar graphs are sparse, such layouts can be obtained in time $\mathcal{O}(n \log n)$ (Lipton et al., 1979). See Figure 4.6 for an example.

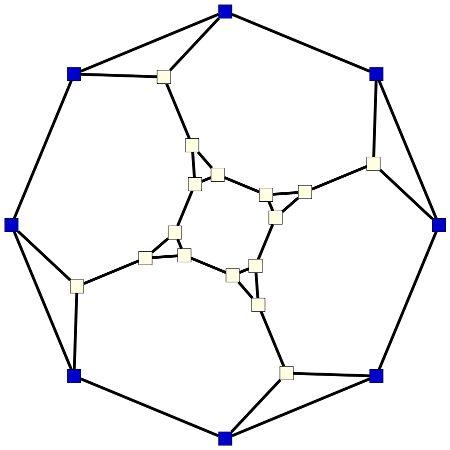


Fig. 4.6. Barycentric layout (darker vertices have been fixed in advance).

Davidson and Harel (1996) deviate even further from models of physical reality. They define attraction and repulsion potentials

$$U_{\mathsf{attr}}(p_u, p_v) = c_{\mathsf{attr}} \cdot \|p_u - p_v\|^2$$
$$U_{\mathsf{rep}}(p_u, p_v) = \frac{c_{\mathsf{rep}}}{\|p_u - p_v\|^2}$$

that are similar to the respective forces of Fruchterman and Reingold (1991). Note that the attraction potential is a scaled version of the zero-length spring potential of Tutte (1963). Combining attraction and repulsion yields a spring potential $U_{\mathsf{spring}}(p_u, p_v) = U_{\mathsf{attr}}(p_u, p_v) + U_{\mathsf{rep}}(p_u, p_v)$ that is again computed for pairs of adjacent vertices. Note that this definition results in an ideal edge length of $\sqrt[4]{c_{\mathsf{rep}}/c_{\mathsf{attr}}}$. Generalizing the repulsion analogy, they also define potentials penalizing vertices that lie close to the boundary of the layout area, and potentials penalizing short distances between a vertex and an edge.

A distinct ingredient of the objective function used in Davidson and Harel (1996) is a potential weighting of the number of crossings in the current layout. Crossings tend to be displeasing and hindering to a viewer (Purchase et al., 1997), and their minization is thus a reasonable criterion for good layout. However, counting crossings leads to a discrete objective function that can no longer be treated by an algorithm based on gradient methods. Moreover, minimizing the number of crossings is also an $\mathcal{NP}$-hard problem (Garey and Johnson, 1983). To obtain at least a local minimum of U_{DH}, *simulated annealing* (see Reeves 1995 for a more recent textbook), a general method for minimizing objective functions of combinatorial problems, is used. It is only by coincidence that this method also has a physical analogy. Given a candidate solution, a new solution is proposed by slightly modifying the current one. If the new solution reduces the value of the objective function, it becomes the new candidate solution. Otherwise, it becomes the new candidate solution only with probability $e^{\frac{-\Delta U}{T}}$, where ΔU is the increase of the objective function, and $T > 0$ is the *temperature* parameter controlling the algorithms ability to climb up hills in the energy landscape. Convergence is enforced by slowly lowering T to zero. A more precise description of a typical simulated annealing algorithm used for graph layout is given in Algorithm 7.

Algorithm 7: Simulated annealing

```
Input: graph G = (V, E)
       initial placement p = (p_v)_{v∈V}
Output: placement p with locally optimal value U(p)

while T > THRESHOLD do
    for v ∈ V do
        p^old ← p
        p_v ← p_v + Δ_random
        if U(p^old) < U(p) then
            with probability 1 − e^((U(p^old) − U(p))/T) reset p ← p^old
    reduce T
```

A number of heuristics to speed up layout computation with the notoriously slow simulated annealing is introduced by Tunkelang (1994). Most notably, there is no initial placement, but vertices are introduced in a breadth-first-search order starting in the graph-theoretic center, and positions are restricted to a coarse grid with only few types of displacements allowed. A parallel implementation is described in Monien et al. (1996).

An experimental comparison (Brandenburg et al., 1996) of (sequential) implementations revealed that the approaches presented in these last two sections (excluding the barycentric model) yield comparable layouts. As a very general rule-of-thumb, energy-based placement approaches tend to produce better results for small to medium-sized graphs (around 30 vertices), while force-directed placement approaches are considerably faster (experience suggest that they need only one tenth of the number of iterations).

Besides gradient methods and simulated annealing, genetic algorithms have been applied to physically motivated objective functions (Kosak et al., 1994; Masui, 1992). Branke et al. (1997) use force-directed methods as a local fine-tuning step of their genetic algorithm.

4.4 Modeling with Forces and Energies

Forces and potential energies have been found to model elementary criteria for readable layouts of straight-line representations of graphs. While the results are usually satisfactory with respect to the two criteria mentioned in the introductory section of this chapter (uniform vertex distribution, uniform edge lengths, and, as a consequence, symmetry), such drawings may not be useful for graphs representing specific structural information.

An important aspect that has not been covered yet is the immense expressive power of force-directed or energy-based placement for formulating layout design goals. This section contains a number of examples, showing how forces can be used to formulate criteria for good layout far beyond vertex distribution and edge length.

The following paragraphs sketch modeling ideas from the literature, comprising a toolbox for the adjustment of force-directed or energy-based layout methods to account for a fairly broad range of requirements.

3D Layouts. For some purposes it may be necessary or desirable to represent a graph in three-dimensional space. In combination with interactive (preferably fly-through) browsers, these appear to be particularly useful for exploring large graphs. The methods outlined so far make no particular assumptions on the number of dimensions and are easily modified to produce three-dimensional (or one-dimensional, for that matter) layouts. For example, a straightforward generalization of the method by Frick et al. (1995) is given in Bruß and Frick (1996). The approach of Davidson and Harel (1996) is adjusted by Cruz and Twarog (1996), where the crossing count, superfluous in three dimensions, is substituted with edge-edge repulsion. See Chapter 7 for more on three-dimensional graph layout.

Clustering. Important information may be represented by intrinsic or extrinsic clusterings of vertices. Since experimental work suggests that users tend to group geometrically close vertices (McGrath et al., 1996), it seems desirable to keep vertices of the same cluster close to each other, while those in different clusters should be further apart.

By their very nature, spring-type methods geometrically cluster dense subgraphs. While this is not at all true for other definitions of clusters, these can be reduced to dense subgraphs by introducing dummy vertices that represent the clusters, and connecting them appropriately (Eades et al., 1997a; Huang and Eades, 1998a). For each cluster, a new vertex is introduced, attracting its contained vertices, while repelling vertices of other clusters. To achieve larger distances between clusters, additional repelling forces are introduced between dummy vertices and all other vertices not contained in their respective cluster. Figure 4.8(a) gives a small example.

Instead of introducing dummy vertices to represent cluster centroids, a *meta-graph* representing connections between clusters is introduced by Wang and Miyamoto (1996). It consists of a vertex for each cluster, and has an

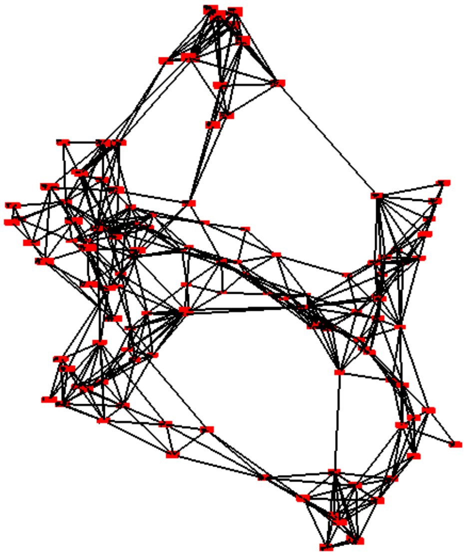


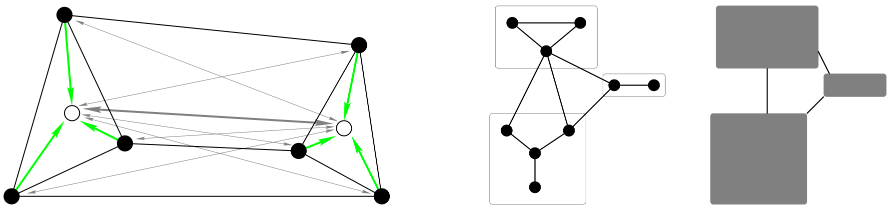
Fig. 4.7. 3D layout of travel distances between 31 U.S. cities (Bruß and Frick, 1996).

edge between each pair of distinct meta-vertices containing adjacent vertices of the original graph. To provide the necessary area for clusters, meta-vertices are not assumed to have point size, but to occupy rectangular areas. Consequently, first a technique is introduced that accounts for the fact that actual edge lengths depend on the relative positions of rectangularly shaped vertices. In the placement iteration, contribution weights are used to gradually shift from an emphasis of forces in the meta-graph (for cluster spacing) to forces in the input graph. More detail on force-directed methods to represent clusters is provided in Chapter 8.

Directed edges. For directed graphs such as data-flow or time-dependency graphs, it is often desirable to have the directed edges point into roughly the same direction. Using forces exerted on edges by external fields of a chosen orientation, edges can be pushed to point in any given direction (Sugiyama and Misue, 1995). Let θ be the angle between an edge's prescribed direction and its current direction, and let $\overrightarrow{p_u p_v}^{\perp}$ be the unit length vector perpendicular to $\overrightarrow{p_u p_v}$ and pointing towards a decrease of θ, then rotative forces

$$f_{\mathsf{rot}}(p_u, p_v) = b \cdot ||p_u - p_v||^{c_1} \cdot \theta^{c_2} \cdot \overrightarrow{p_u p_v}^{\perp}$$

can be added to the net force vector, thus pressing the edge to reduce θ (see Figure 4.9). While b is a constant that controls the strength of the magnetic field acting on an edge between u and v with respect to the other forces, c_1 and c_2 are parameters that control the relative dependency of rotative forces on vertex distance and angle deviation, respectively.



(a) Dummy vertices represent the centroids of clusters (Eades et al., 1997a; Huang and Eades, 1998a).

(b) A clustering with its corresponding meta-graph (Wang and Miyamoto, 1996).

Fig. 4.8. Force-directed models for clusters.

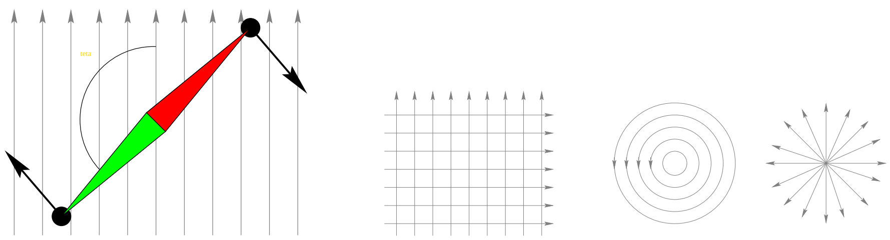


Fig. 4.9. Magnetic springs represent directed edges subject to an arbitrary force field (Sugiyama and Misue, 1995).

Curved edges. So far, all representations considered here use straight lines to depict edges. It is shown in Brandes and Wagner (1998b) how the layout of curved line representations can be reduced to the straight-line case by placing control points instead of vertices. More precisely, edges that are to be depicted by Bézier curves are replaced with vertices representing their control points. Then an auxiliary graph is constructed by introducing a dummy edge of appropriate desired length for each control segment, and additional attractive and repulsive forces to ensure certain properties of the resulting layout. Figure 4.10 shows a graph of train connections with curved representation of edges that are classified as "transitive" (top left), the auxiliary graph constructed for this instance (bottom left), and a larger example (right). The same technique obviously works for polyline representations.

Dynamic layout. When users look at the drawing of a graph, they familiarize themselves with this drawing by building a cognitive representation called the *mental map*. Dynamic graph layout is concerned with the trade-off between static layout quality and the user's mental map.

A framework to extend static graph layout methods to dynamic ones by incorporating a difference metric into the objective function is advocated

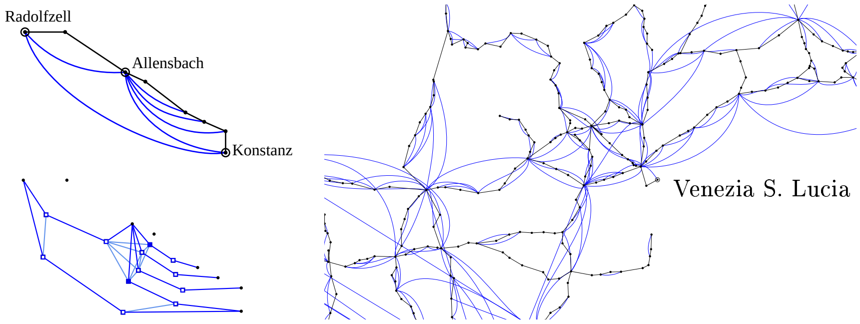


Fig. 4.10. Curved edge layout using an auxiliary graph (Brandes and Wagner, 1998b).

in Brandes and Wagner (1997). For energy-based methods, the difference metric summing over the Euclidean distances of vertex positions translates nicely into an attraction potential for imaginary springs of natural length zero, which can be used in force-directed approaches as well. An overview of approaches for drawing dynamic and interactive graphs is provided in Chapter 9.

Constraints. Constraints can be integrated into force-directed or energy-based placement methods in several ways, depending on the type of constraint. Unary constraints, i.e., restrictions on the admissible positions of single vertices, are easily satisfied if feasible positions form a connected region in the layout area or space (see Figure 4.11). In this case, every configuration remains reachable, and position updates can be modified to ensure that each vertex is at a feasible position at all times.

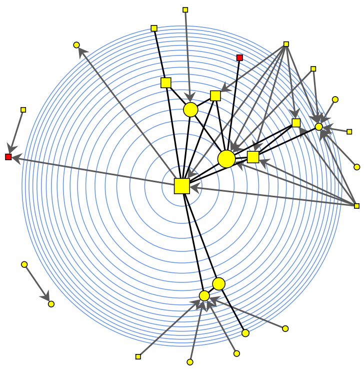


Fig. 4.11. Unary constraints. In this drawing of a social network, each vertex is restricted to lie on the circumference of a circle, signifying its centrality (Brandes, 1999).

A second way to satisfy unary constraints that also handles disconnected feasible regions is to allow vertices to temporarily occupy infeasible positions, but to use additional forces tht drag vertices to feasible positions or penalties pushing them away from infeasible positions, respectively. The penalty approach is easily extended to arbitrary constraints, but rarely guarantees satisfaction of all constraints in practice.

To satisfy pairwise linear (in)equalities, the placement iteration is interleaved with conflict detection in Wang and Miyamoto (1996). Every pair of vertices that would not satisfy a constraint if a displacement is carried out, is joined by a "rigid stick" that forces them to move together while maintaining the constraint. Note that criteria corresponding to weak linear pairwise constraints can be modeled using the magnetic springs of Sugiyama and Misue (1995).

Arbitrary constraints can be incorporated in methods based on physical analogies by applying more elaborate constraint satisfaction techniques, as in He and Marriott (1998). A survey of the use of constraints in graph layout is given by Tamassia (1998)

From a more general point of view, the above modeling techniques can be seen as instances of a generic framework for layout models, in which arbitrary layout variables are assigned values subject to some constraints and criteria that describe feasible and desirable configurations. Graph layout is thus cast as a general constraint optimization problem, for which objective functions can be devised by combining criteria for small configurations like pairs of adjacent or non-adjacent vertices, triples of connected vertices, pairs of edges, and so on. Force-directed and energy-based methods fit well into this framework, because they are intuitive and incorporate an implicit notion of compromise between the various criteria.

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